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Nonanticommutative $U(1)$ SYM theories: renormalization, fixed points and infrared stability

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ABSTRACT: Renormalizable nonanticommutative SYM theories with chiral matter in the adjoint representation of the gauge group have been recently constructed in [arXiv:0901.3094]. In the present paper we focus on the $U_*(1)$ case with matter interacting through a cubic superpotential. For a single flavor, in a superspace setup and manifest background covariant approach we perform the complete one-loop renormalization and compute the beta-functions for all couplings appearing in the action. We then generalize the calculation to the case of $SU(3)$ flavor matter with a cubic superpotential viewed as a nontrivial NAC generalization of the ordinary abelian $N = 4$ SYM and its marginal deformations. We find that, as in the ordinary commutative case, the NAC $N = 4$ theory is one-loop finite. We provide general arguments in support of all-loop finiteness. Instead, deforming the superpotential by marginal operators gives rise to beta-functions which are in general non-vanishing. We study the spectrum of fixed points and the RG flows. We find that nonanticommutativity always makes the fixed points unstable.

KEYWORDS: Supersymmetric gauge theory, Superspaces, Non-Commutative Geometry

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1 Introduction

Supersymmetric field theories can be defined on a nonanticommutative (NAC) superspace [1–5] where the spinorial variables satisfy $\{\theta^\alpha, \theta^\beta\} = \mathcal{F}^{\alpha\beta}$. The nontrivial spinorial algebra usually breaks supersymmetry down to $N = 1/2$. The tensor $\mathcal{F}^{\alpha\beta}$ has a stringy origin as the graviphoton field which appears in the $N = 2$ supergravity multiplet when taking the zero string length limit.

NAC deformations of supersymmetric field theories have been extensively studied in four [6]–[16] and lower [17] dimensions. In particular, since supersymmetry is partially broken a mandatory question is whether these theories maintain the robust renormalizability properties of their parent anticommutative theories. To this respect all NAC field theories investigated so far have two common features: 1) Nonanticommutativity is a mechanism of soft susy breaking; 2) Renormalizable NAC theories are not obtained from their ordinary parents by simply promoting products to NAC products in the original action but always require the addition of extra soft terms.

One of the main issues to be addressed is the NAC formulation of gauge theories in interaction with chiral matter. Recently, a renormalizable NAC deformation of SYM theories with matter in the adjoint representation of the gauge group has been proposed [18]. This opens the possibility of investigating NAC deformations of SYM theories with extended supersymmetry. In particular, quantum consistent NAC deformations of $N = 4$ SYM are now available which provide the low energy dynamics of a stack of D3-branes in the presence of a non-vanishing RR two-form. This is an indispensable ingredient for generalizing the AdS/CFT correspondence to backgrounds with RR forms turned on in the directions parallel to the branes.

As discussed in [18] for the $SU(\mathcal{N}) \otimes U(1)$ gauge group, adding adjoint matter to a NAC SYM theory with a non-trivial cubic superpotential leads to a theory which is not simply the natural generalization of the corresponding ordinary one obtained by promoting products to $*$ -products in the classical action. In fact, the strict interplay between renormalizability and gauge invariance requires to assign a different coupling constant to the quadratic term for the abelian matter superfields in order to tune the renormalization of the abelian fields with the one for the non-abelians. This opens the possibility to add a renormalizable, $N = 1/2$ and gauge invariant cubic superpotential. Moreover, it changes the gauge-matter coupling in vertices where abelian (anti)chirals are present. As a crucial consequence, the evaluation of one-loop diagrams reveals that only $N = 1/2$ susy and supergauge invariant divergent structures get produced. Therefore, a one-loop renormalizable action is obtained by adding all possible soft susy-breaking and supergauge invariant couplings allowed by dimensional analysis.

Sufficient evidence for one-loop renormalizability has been given [18], but the complete renormalization has not been carried out yet. In fact, due to the non-trivial group structure, the form of the action is quite complicated and the calculation of all one-loop divergent contributions would imply the evaluation of a large number of diagrams.

In order to avoid technical complications related to the group structure, in this paper we focus on the $U_*(1)$ case. The noncommutative $U_*(1)$ gauge theory is obtained from the non(anti)commutative $U(\mathcal{N})$ theory in the limit $\mathcal{N} \rightarrow 1$. Despite the abelian nature of the generator algebra the resulting gauge theory is highly interacting as a consequence of the non(anti)commutative nature of the $*$ -product.

In this case complications related to the different renormalization undergone by non-abelian and abelian superfields [18] are absent and the general structure of SYM theories with matter in the adjoint representation of the gauge group is rather simpler.

We first consider the case of a single matter superfield interacting with a cubic superpotential. We complete the one-loop renormalization of the theory and compute the corresponding beta-functions.

We then generalize the calculation to the case of three adjoint chiral superfields in interaction through the superpotential

$$\begin{aligned}
 & h_1 \int d^4x d^2\theta \Phi_1 * \Phi_2 * \Phi_3 - h_2 \int d^4x d^2\theta \Phi_1 * \Phi_3 * \Phi_2 \\
 & + \bar{h}_1 \int d^4x d^2\bar{\theta} \bar{\Phi}^1 * \bar{\Phi}^2 * \bar{\Phi}^3 - \bar{h}_2 \int d^4x d^2\bar{\theta} \bar{\Phi}^1 * \bar{\Phi}^3 * \bar{\Phi}^2
 \end{aligned} \tag{1.1}$$

For $h_1 = h_2, \bar{h}_1 = \bar{h}_2$ it exhibits a global $SU(3)$ invariance and can be interpreted as a nontrivial NAC deformation of the ordinary abelian $N = 4$ SYM theory. Turning on nonanticommutativity breaks $N = 4$ to $N = 1/2$. More generally, for $h_1 \neq h_2$ and/or $\bar{h}_1 \neq \bar{h}_2$ the $SU(3)$ symmetry is lost and the superpotential (1.1) describes the NAC generalization of a marginally deformed [19, 20] $N = 4$ SYM theory.

We find that at one-loop the theory with equal couplings is *finite* exactly like the ordinary $N = 4$ counterpart. Using perturbative arguments based on dimensional considerations and symmetries of the theory we provide evidence that the theory should be

finite at all loop orders. On the other hand, in the presence of marginal deformations UV divergences arise which in general prevent the theory from being at a fixed point.

Both for the one and three-flavor cases we study the spectrum of fixed points and the RG flows in the parameter space. We find that nonanticommutativity always renders the fixed points IR and UV unstable. Compared to the ordinary case, we lose the IR stability of the fixed point corresponding to the free theory ($h = \bar{h} = 0$ and $h_1 = h_2, \bar{h}_1 = \bar{h}_2$). This is due to the fact that in the NAC case the parameter space gets enlarged and new directions appear which drive the theories away from the fixed point.

The organization of the paper is the following: in section 2 we define $U_*(1)$ NAC SYM theories with one and three chiral superfields in the adjoint representation of the gauge group, we discuss their gauge invariance and write their renormalizable actions according to the results of [18]. In section 3 we present the one-loop renormalization for the case of a single matter field and the corresponding beta-functions. The same is done for the case of three chiral superfields in section 4. Finally, in section 5 we discuss the spectra of fixed points and their stability. Conclusions follow plus an appendix where all technical details required by the calculations are collected.

2 $U_*(1)$ NAC SYM theories

$N = (\frac{1}{2}, 0)$ NAC superspace is spanned by nonanticommutative coordinates $(x^{\alpha\dot{\alpha}}, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ satisfying

$$\{\theta^\alpha, \theta^\beta\} = 2\mathcal{F}^{\alpha\beta} \quad \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0 \quad [x^{\alpha\dot{\alpha}}, x^{\beta\dot{\beta}}] = [x^{\alpha\dot{\alpha}}, \theta^\beta] = [x^{\alpha\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}] = 0 \quad (2.1)$$

where $\mathcal{F}^{\alpha\beta}$ is a 2×2 symmetric, constant matrix. This algebra is consistent only in euclidean signature where the chiral and antichiral sectors are totally independent and not related by complex conjugation.

The class of smooth superfunctions on the NAC superspace is endowed with the NAC but associative product

$$\phi * \psi \equiv \phi e^{-\overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta} \psi = \phi \psi - \phi \overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta \psi - \frac{1}{2} \mathcal{F}^2 \partial^2 \phi \partial^2 \psi \quad (2.2)$$

where $\mathcal{F}^2 \equiv \mathcal{F}^{\alpha\beta} \mathcal{F}_{\alpha\beta}$. (Anti)chiral superfields can be consistently defined by the constraints $\overleftarrow{D}_{\dot{\alpha}} * \phi = D_\alpha * \bar{\phi} = 0$, where in chiral representation $D_\alpha = \partial_\alpha + i\bar{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}$ and $\overleftarrow{D}_{\dot{\alpha}} = \overleftarrow{\partial}_{\dot{\alpha}}$ (we use conventions of [21]).

$U_*(1)$ supergauge group is defined as the limit of the NAC $U(\mathcal{N})$ group when $\mathcal{N} = 1$. Its elements are the chiral and antichiral superfields

$$g(x, \theta, \bar{\theta}) = e_*^{i\Lambda(x, \theta, \bar{\theta})}, \quad \bar{g}(x, \theta, \bar{\theta}) = e_*^{i\bar{\Lambda}(x, \theta, \bar{\theta})} \quad (2.3)$$

which satisfy a noncommutative algebra.

Given the non-abelian nature of $U_*(1)$ an adjoint representation can be defined according to the following prescription: a chiral superfield ϕ belongs to the adjoint representation of the gauge group if under supergauge transformations it transforms as

$$\phi \rightarrow \phi' = e_*^{i\Lambda} * \phi * e_*^{-i\Lambda} \quad (2.4)$$

Equivalently, the transformation law for an antichiral superfield $\bar{\phi}$ in the adjoint representation reads

$$\bar{\phi} \rightarrow \bar{\phi}' = e_*^{i\bar{\Lambda}} * \bar{\phi} * e_*^{-i\bar{\Lambda}} \quad (2.5)$$

As in the ordinary non-abelian case, supersymmetric $U_*(1)$ NAC Yang-Mills theories can be described in a manifestly covariant way by introducing a scalar prepotential V in the adjoint representation of the gauge group transforming as

$$e_*^V \rightarrow e_*^{V'} = e_*^{i\bar{\Lambda}} * e_*^V * e_*^{-i\bar{\Lambda}} \quad (2.6)$$

Being the theory in euclidean signature, V has to be *pure imaginary*, $V^\dagger = -V$.

We define gauge covariant derivatives in superspace in the so-called *gauge antichiral* representation [21] as

$$\nabla_A \equiv (\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\alpha\dot{\alpha}}) = (D_\alpha, e_*^V * \bar{D}_{\dot{\alpha}} e_*^{-V}, -i\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\}_*) \quad (2.7)$$

They act on superfields in the adjoint representation according to the prescription

$$\nabla * A \equiv [\nabla, A]_* = (DA) - i[\Gamma, A]_* \quad (2.8)$$

where the connections are explicitly given by

$$\Gamma_\alpha = 0, \quad \bar{\Gamma}_{\dot{\alpha}} = ie_*^V * \bar{D}_{\dot{\alpha}} e_*^{-V}, \quad \bar{\Gamma}_{\alpha\dot{\alpha}} = -iD_\alpha \bar{\Gamma}_{\dot{\alpha}} \quad (2.9)$$

The corresponding field strengths are defined as *-commutators of supergauge covariant derivatives

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{2}[\nabla^\alpha, \bar{\nabla}_{\alpha\dot{\alpha}}]_*, \quad \tilde{W}_\alpha = -\frac{1}{2}[\bar{\nabla}^{\dot{\alpha}}, \bar{\nabla}_{\alpha\dot{\alpha}}]_* \quad (2.10)$$

and satisfy the Bianchi's identities $\nabla^\alpha * \tilde{W}_\alpha + \bar{\nabla}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} = 0$. In terms of gauge connections they are given by

$$\bar{W}_{\dot{\alpha}} = \frac{i}{2}D^\alpha \bar{\Gamma}_{\alpha\dot{\alpha}} = D^2 \bar{\Gamma}_{\dot{\alpha}}, \quad \tilde{W}_\alpha = \frac{i}{2}\partial_\alpha^{\dot{\alpha}} \bar{\Gamma}_{\dot{\alpha}} + \frac{i}{2}[\bar{\nabla}^{\dot{\alpha}}, \bar{\Gamma}_{\alpha\dot{\alpha}}]_* \quad (2.11)$$

Covariantly (anti)chiral superfields can be defined according to $[\bar{\nabla}_{\dot{\alpha}}, \Phi]_* = 0$ and $[\nabla_\alpha, \bar{\Phi}]_* = 0$, respectively.

Specializing the results of [18] to the $U_*(1)$ case the most general renormalizable action for a NAC SYM theory with one self-interacting chiral superfield in the adjoint representation of the gauge group is given by (for simplicity we consider massless matter)

$$\begin{aligned} S = & \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} \\ & + \int d^4x d^4\theta \Phi * \bar{\Phi} + h \int d^4x d^2\theta \Phi_*^3 + \bar{h} \int d^4x d^2\bar{\theta} \bar{\Phi}_*^3 \\ & + it_1 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \partial_\alpha^{\dot{\alpha}} \bar{\Gamma}_{\beta\dot{\alpha}} * \Phi * \bar{\Phi} + t_2 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}^{\alpha\dot{\alpha}} * \bar{\Gamma}_{\alpha\dot{\alpha}} * \bar{\Phi}_*^3 \\ & + t_3 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} * \Phi * \bar{\Phi} \\ & + h_3 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi * \nabla^2 \Phi * \nabla^2 \Phi \\ & + h_4 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \nabla^2 \Phi * \Phi * \bar{\Phi}_*^2 + h_5 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi * \bar{\Phi}_*^4 \end{aligned} \quad (2.12)$$

where $\Phi \equiv e_*^V * \phi * e_*^{-V}$, $\bar{\Phi} = \bar{\phi}$ are covariantly (anti)chiral superfields expressed in terms of ordinary (anti)chirals. We choose to indicate explicitly the $*$ -product everywhere without distinguishing the cases where it actually coincides with the ordinary product. For example, it is easy to see that $\int d^4x d^2\bar{\theta} \bar{\Phi}_*^3 = \int d^4x d^2\bar{\theta} \bar{\Phi}^3$ up to superspace total derivatives.

We note that in contrast with the $SU(\mathcal{N}) \otimes U(1)$ case [11] the pure gauge action contains only the NAC generalization of the standard quadratic term. In fact, it is easy to see that all the extra terms which need be taken into account in the $SU(\mathcal{N}) \otimes U(1)$ case for insuring renormalizability and gauge invariance are identically zero in the $U_*(1)$ limit.

More generally, we consider a theory with three different flavors in the (anti)fundamental representation of $SU(3)$, still interacting through a cubic superpotential. Again, using the results of [18] the most general renormalizable action which respects two global $U(1)$ symmetries is

$$\begin{aligned}
 S = & \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} + \int d^4x d^4\theta \Phi_i * \bar{\Phi}^i \\
 & + \int d^4x d^2\theta (h_1 \Phi_1 * \Phi_2 * \Phi_3 - h_2 \Phi_1 * \Phi_3 * \Phi_2) \\
 & + \int d^4x d^2\bar{\theta} (\bar{h}_1 \bar{\Phi}^1 * \bar{\Phi}^2 * \bar{\Phi}^3 - \bar{h}_2 \bar{\Phi}^1 * \bar{\Phi}^3 * \bar{\Phi}^2) \\
 & + it_1 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \partial_{\alpha}^{\dot{\alpha}} \bar{\Gamma}_{\beta\dot{\alpha}} * \Phi_i * \bar{\Phi}^i + t_2 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}^{\alpha\dot{\alpha}} * \bar{\Gamma}_{\alpha\dot{\alpha}} * \bar{\Phi}^1 * \bar{\Phi}^2 * \bar{\Phi}^3 \\
 & + t_3 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} * \Phi_i * \bar{\Phi}^i \\
 & + \tilde{h}_3 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \nabla_{\alpha} \Phi_1 * \nabla_{\beta} \Phi_2 * \Phi_3 + h_3 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi_1 * \nabla^2 \Phi_2 * \nabla^2 \Phi_3 \\
 & + h_4^{(=)} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \sum_{i=1}^3 \nabla^2 \Phi_i * \Phi_i * \bar{\Phi}^i * \bar{\Phi}^i \\
 & + h_4^{(\neq)} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \sum_{i<j} \nabla^2 \Phi_i * \Phi_j * \bar{\Phi}^i * \bar{\Phi}^j \\
 & + h_5 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi_i * \bar{\Phi}^i * \bar{\Phi}^1 * \bar{\Phi}^2 * \bar{\Phi}^3
 \end{aligned} \tag{2.13}$$

in terms of covariantly (anti)chiral superfields $\Phi_i, \bar{\Phi}^i$. We note that one extra coupling \tilde{h}_3 is allowed in this case which would be trivially zero in the action (2.12), for symmetry reasons. The two global $U(1)$ charges for the matter superfields are $(1, -1, 0)$ and $(0, 1, -1)$ respectively, whereas antichiral superfields carry opposite charges.

The two actions are invariant under the following gauge transformations

$$\begin{aligned}
 \delta \Phi_i &= i[\bar{\Lambda}, \Phi_i]_*, & \delta \bar{\Phi}^i &= i[\bar{\Lambda}, \bar{\Phi}^i]_* \\
 \delta \bar{\Gamma}_{\alpha\dot{\alpha}} &= [\bar{\nabla}_{\alpha\dot{\alpha}}, \bar{\Lambda}]_*, & \delta \bar{W}_{\dot{\alpha}} &= i[\bar{\Lambda}, \bar{W}_{\dot{\alpha}}]_*
 \end{aligned} \tag{2.14}$$

We note that except for the transformation of $\bar{\Gamma}$ the right hand sides vanish when $\mathcal{F}^{\alpha\beta} = 0$, as it should in the ordinary $U(1)$ case (when taking the commutative limit matter in the adjoint representation of $U_*(1)$ is mapped into $U(1)$ singlets).

In general, the cubic superpotential of (2.13) is a function of four independent couplings $h_1, h_2, \bar{h}_1, \bar{h}_2$. If we set $h_1 = h_2$ and $\bar{h}_1 = \bar{h}_2$ the action (2.13) has a global SU(3) invariance which can be thought of as related to the R-symmetry of an ordinary $N = 4$ SYM theory. Therefore, we study the theory (2.13) as a non-trivial NAC deformation of the abelian $N = 4$ SYM.¹ We note that, while the ordinary U(1) $N = 4$ theory is a free theory of one vector superfield plus three chiral gauge singlets in the fundamental of SU(3), the NAC deformation we propose is highly interacting.

More generally, if we set $h_1 = he^{i\pi\beta}, h_2 = he^{-i\pi\beta}$ and $\bar{h}_1 = \bar{h}e^{-i\pi\bar{\beta}}, \bar{h}_2 = \bar{h}e^{i\pi\bar{\beta}}$ only the two global U(1)'s survive and we have the NAC generalization of beta-deformed theories [20]. We note that, being the theory in euclidean space with strictly real matter superfields, we need take the deformation parameters $\beta, \bar{\beta}$ to be pure imaginary in order to guarantee the reality of the action. In the ordinary anticommutative case supersymmetric theories with pure imaginary β have been studied in [23].

Both in the $N = 4$ case and in its less supersymmetric marginal deformations, supersymmetry is broken to $N = 1/2$ by the NAC superspace structure.

In order to perform perturbative calculations we use background field method [21] suitably generalized to the NAC superspace [11]. As a result, at any given order in the loop expansion the contributions to the effective action are expressed directly in terms of covariant derivatives and field strengths without any explicit dependence on the prepotential V .

We split the Euclidean prepotential as $e_*^V \rightarrow e_*^V * e_*^U$ where U is the background prepotential and V its quantum counterpart. Consequently, the covariant derivatives (2.7) become

$$\nabla_\alpha = \nabla_\alpha = D_\alpha, \quad \bar{\nabla}_{\dot{\alpha}} = e_*^V * \bar{\nabla}_{\dot{\alpha}} * e_*^{-V} = e_*^V * (e_*^U * \bar{D}_{\dot{\alpha}} * e_*^{-U}) * e_*^{-V} \quad (2.15)$$

Covariantly (anti)chiral superfields in the adjoint representation are expressed in terms of background covariantly (anti)chiral objects as

$$\bar{\Phi} = \bar{\Phi}, \quad \Phi = e_*^V * \Phi * e_*^{-V} = e_*^V * (e_*^U * \phi * e_*^{-U}) * e_*^{-V} \quad (2.16)$$

We then split $\Phi \rightarrow \Phi + \Phi_q$ and $\bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_q$, where $\Phi, \bar{\Phi}$ are background fields and $\Phi_q, \bar{\Phi}_q$ their quantum fluctuations.

We break the invariance under quantum gauge transformations [11, 21] by choosing gauge-fixing functions $f = \bar{\nabla}^2 * V, \bar{f} = \nabla^2 * V$, while preserving manifest invariance of the effective action and correlation functions under background gauge transformations [11, 21].

The ghost action associated to the gauge-fixing is given in terms of background covariantly (anti)chiral FP and NK ghost superfields as

$$S_{\text{gh}} = \int d^4x d^4\theta \left[\bar{c}'c - c'\bar{c} + \dots + \bar{b}b \right] \quad (2.17)$$

In ref. [11] the gauge-fixing procedure for NAC gauge theories has been discussed in detail. For the present scopes in the appendix we summarize the procedure and collect the Feynman rules necessary for one-loop calculations.

¹At classical level, the NAC generalization of $N = 4$ SYM theories has been studied in [22] starting from an action which is the ordinary $N = 4$ action with products promoted to *-products.

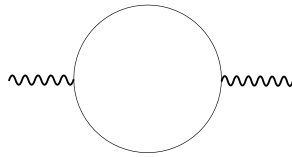


Figure 1. Gauge self-energy diagram.

We work in dimensional regularization, $n = 4 - 2\epsilon$. All divergent integrals are expressed in terms of the self-energy integral

$$\mathcal{S} = \int d^n q \frac{1}{q^2(q-p)^2} = \frac{1}{(4\pi)^2} \frac{1}{\epsilon} + \mathcal{O}(1) \quad (2.18)$$

3 One flavor case: renormalization and β -functions

We first concentrate on the theory described by the action (2.12) and perform one-loop renormalization.

Using Feynman rules listed in the appendix we draw all possible one-loop divergent diagrams. A useful selection rule arises by looking at the overall power of the NAC parameter for a given diagram. In fact, as it is clear from the dimensional analysis of refs. [11, 18] divergent contributions can be at most quadratic in $\mathcal{F}^{\alpha\beta}$. Since powers of \mathcal{F} come from vertices and from the expansion of covariant propagators (see eqs. (A.14), (A.30)) it is easy to count the overall power of the NAC parameter and withdraw diagrams with too many \mathcal{F} 's.

According to standard D -algebra arguments, in the NAC case as in the ordinary one divergent contributions to the gauge effective action come only from diagrams with a chiral matter/ghost quantum loop [11]. For the $U_*(1)$ theory the only potentially divergent contribution comes from the two-point diagram in figure 1 with interaction vertices arising from the expansion (A.14) of the covariant chiral propagator. Being the vertices of order \mathcal{F} the result would be of order \mathcal{F}^2 . Since dimensional analysis does not allow for self-energy divergent contributions proportional to the NAC parameter we expect the divergent part of this diagram to vanish. In fact, by direct inspection it is easy to see that after D -algebra it reduces to a tadpole thus giving a vanishing contribution in dimensional regularization. Therefore, the gauge action does not receive any one-loop contributions. This is consistent with the result of [11] specialized to the $\mathcal{N} = 1$ case.

We then concentrate on the renormalization of the gauge/matter part of the action (2.12). Using Feynman rules in the appendix we select diagrams in figures 2, 3 as the only one-loop divergent diagrams. Diagrams (2a, 2c, 2d, 2e) are obtained from diagram (3a) by expanding $1/\square_{\text{cov}}$ as in (A.14) and writing $\overline{W} \sim D\overline{\Gamma}$. All internal lines are associated to ordinary $1/\square$ propagators.

By direct calculation it turns out that diagrams (2d) and (2e) cancel one against the other whereas the rest, after performing D -algebra, leads to the following one-loop effective

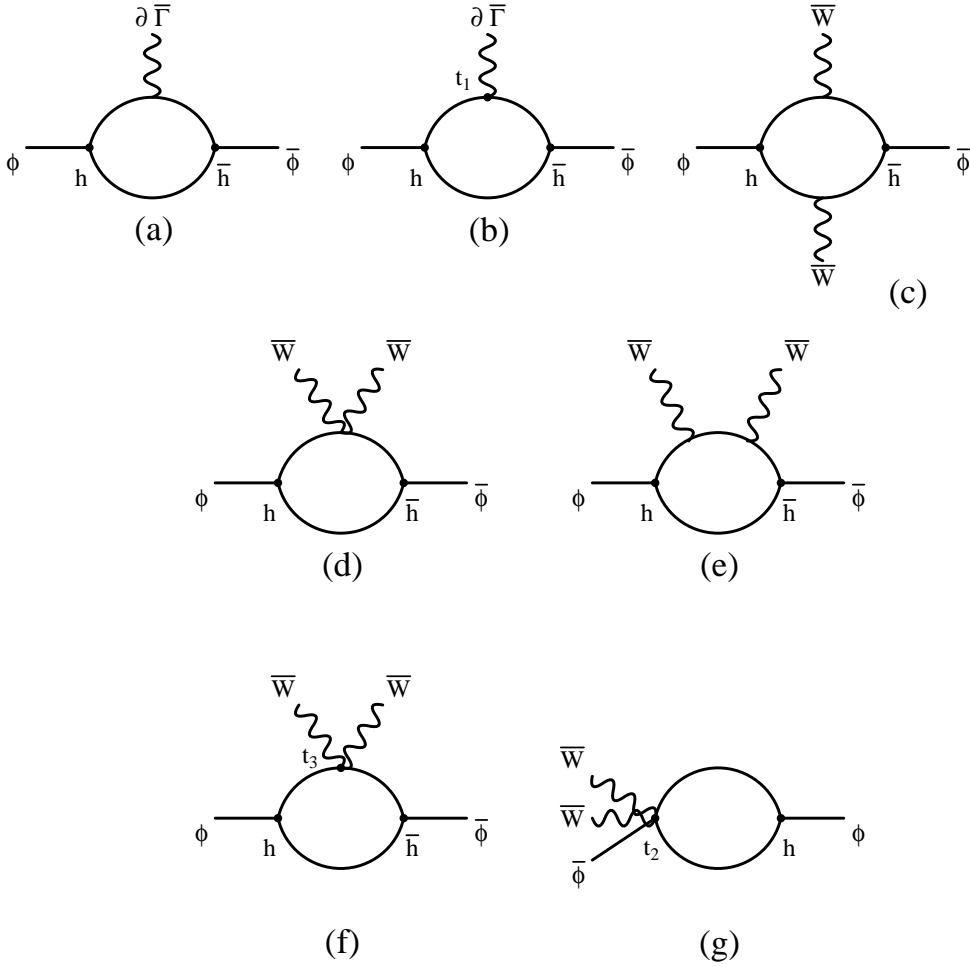


Figure 2. One-loop diagrams contributing to the mixed sector.

action

$$\begin{aligned}
 \Gamma_{\text{div}}^{(1)} = & \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{\mathbf{W}}^{\dot{\alpha}} * \bar{\mathbf{W}}_{\dot{\alpha}} + \int d^4x d^4\theta \Phi * \bar{\Phi} [1 + 18h\bar{h}S] \\
 & + h \int d^4x d^2\theta \Phi_*^3 + \bar{h} \int d^4x d^2\bar{\theta} \bar{\Phi}_*^3 \\
 & + i\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \partial_{\alpha}^{\dot{\alpha}} \bar{\Gamma}_{\beta\dot{\alpha}} * \Phi * \bar{\Phi} [t_1 + 36(h\bar{h} - h\bar{h}t_1)S] \\
 & + t_2 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}^{\alpha\dot{\alpha}} * \bar{\Gamma}_{\alpha\dot{\alpha}} * \bar{\Phi}_*^3 \\
 & + \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\mathbf{W}}^{\dot{\alpha}} * \bar{\mathbf{W}}_{\dot{\alpha}} * \Phi * \bar{\Phi} [t_3 + 36(h\bar{h} - h\bar{h}t_3 - ht_2)S] \\
 & + \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi * (\nabla^2 \Phi)_*^2 [h_3 + (12g^2h - 12g^2t_1h + 3g^2t_1^2h + 6hh_4)S] \\
 & + \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \nabla^2 \Phi * \Phi * \bar{\Phi}_*^2 [h_4 + (72h\bar{h}g^2t_1 - 36h\bar{h}g^2t_1^2 + 648h_3h\bar{h}^2 \\
 & \quad + 324h^2\bar{h}^2 - 144h\bar{h}h_4 + 36hh_5)S] \\
 & + \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi * \bar{\Phi}_*^4 [h_5 + (108h\bar{h}^2g^2t_1^2 + 216h\bar{h}^2h_4 - 144h\bar{h}h_5)S]
 \end{aligned}
 \tag{3.1}$$

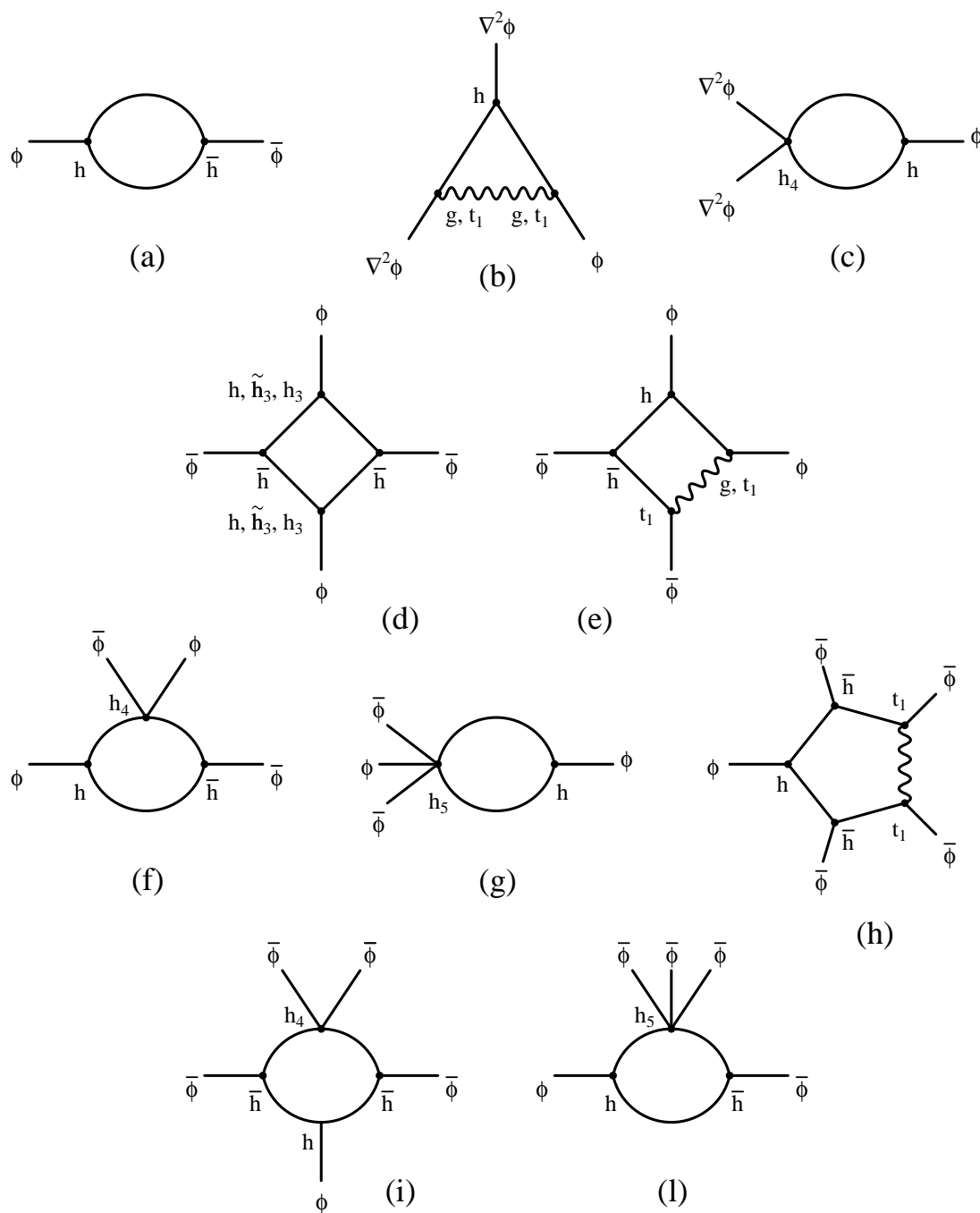


Figure 3. One-loop diagrams contributing to the matter sector.

where \mathcal{S} is given in (2.18).

Few comments are in order. First of all we note that the matter quadratic term does not receive gauge contributions. This is consistent with the results of ref. [18] where it was already shown that the abelian gauge quadratic term does not correct by terms proportional to g^2 . The superpotential does not renormalize thanks to the non-renormalization theorem which holds also in the NAC case. A similar behavior is exhibited by the t_2 -term which, at least at one-loop, seems to be protected from renormalization. However, in this case we do not have any argument for expecting such a protection beyond one-loop.

We now proceed to the renormalization of the theory by defining renormalized coupling constants as

$$\begin{aligned}
 \Phi &= Z^{-\frac{1}{2}} \Phi_B & \bar{\Phi} &= \bar{Z}^{-\frac{1}{2}} \bar{\Phi}_B & & (3.2) \\
 g &= \mu^{-\epsilon} Z_g^{-1} g_B & h &= \mu^{-\epsilon} Z_h^{-1} h_B & \bar{h} &= \mu^{-\epsilon} Z_{\bar{h}}^{-1} \bar{h}_B \\
 t_1 &= Z_{t_1}^{-1} t_{1B} & t_2 &= \mu^{-\epsilon} Z_{t_2}^{-1} t_{2B} & t_3 &= Z_{t_3}^{-1} t_{3B} \\
 h_3 &= \mu^{-\epsilon} Z_{h_3}^{-1} h_{3B} & h_4 &= \mu^{-2\epsilon} Z_{h_4}^{-1} h_{4B} & h_5 &= \mu^{-3\epsilon} Z_{h_5}^{-1} h_{5B}
 \end{aligned}$$

where powers of the renormalization mass μ have been introduced in order to deal with dimensionless renormalized couplings. In order to cancel the divergences in (3.1) we set

$$\begin{aligned}
 Z &= \bar{Z} = 1 - 18 \frac{h\bar{h}}{(4\pi)^2} \frac{1}{\epsilon} \\
 Z_h h &= h + 27 \frac{h^2 \bar{h}}{(4\pi)^2} \frac{1}{\epsilon} \equiv h + \frac{h^{(1)}}{\epsilon} \\
 Z_{\bar{h}} \bar{h} &= \bar{h} + 27 \frac{h\bar{h}^2}{(4\pi)^2} \frac{1}{\epsilon} \equiv \bar{h} + \frac{\bar{h}^{(1)}}{\epsilon} \\
 Z_{h_3} h_3 &= h_3 + \frac{27 h\bar{h} h_3 - 12 g^2 h + 12 g^2 h t_1 - 3 g^2 h t_1^2 - 6 h h_4}{(4\pi)^2} \frac{1}{\epsilon} \equiv h_3 + \frac{h_3^{(1)}}{\epsilon} \\
 Z_{t_1} t_1 &= t_1 + 18 (3 t_1 - 2) \frac{h\bar{h}}{(4\pi)^2} \frac{1}{\epsilon} \equiv t_1 + \frac{t_1^{(1)}}{\epsilon} \\
 Z_{t_2} t_2 &= t_2 + \frac{27 t_2 h\bar{h}}{(4\pi)^2} \frac{1}{\epsilon} \equiv t_2 + \frac{t_2^{(1)}}{\epsilon} \\
 Z_{t_3} t_3 &= t_3 + \frac{54 t_3 h\bar{h} - 36 h\bar{h} + 36 h t_2}{(4\pi)^2} \frac{1}{\epsilon} \equiv t_3 + \frac{t_3^{(1)}}{\epsilon} \\
 Z_{h_4} h_4 &= h_4 + \frac{180 h\bar{h} h_4 - 36 h h_5 + 36 h\bar{h} g^2 t_1^2 - 72 h\bar{h} g^2 t_1 - 648 h_3 h\bar{h}^2 - 324 (h\bar{h})^2}{(4\pi)^2} \frac{1}{\epsilon} \\
 &\equiv h_4 + \frac{h_4^{(1)}}{\epsilon} \\
 Z_{h_5} h_5 &= h_5 - \frac{108 h\bar{h}^2 g^2 t_1^2 + 216 h\bar{h}^2 h_4 - 189 h\bar{h} h_5}{(4\pi)^2} \frac{1}{\epsilon} \equiv h_5 + \frac{h_5^{(1)}}{\epsilon} & (3.3)
 \end{aligned}$$

We have chosen to renormalize the chiral and the antichiral superfields in the same way, although this is not forced by any symmetry of the theory. We note that divergences can be cancelled without renormalizing the NAC parameter $\mathcal{F}^{\alpha\beta}$. Therefore, the star product does not get deformed by quantum corrections.

We compute the beta-functions according to the general prescription

$$\beta_{\lambda_j} = -\epsilon \alpha_j \lambda_j - \alpha_j \lambda_j^{(1)} + \sum_i \left(\alpha_i \lambda_i \frac{\partial \lambda_j^{(1)}}{\partial \lambda_i} \right) \quad (3.4)$$

where λ_j stands for any coupling of the theory and α_j is its naive dimension. Reading the single pole coefficients $\lambda_j^{(1)}$ in eq. (3.3) we finally obtain

$$\begin{aligned}
 \beta_g &= 0 \\
 \beta_h &= \frac{54 h^2 \bar{h}}{(4\pi)^2} \\
 \beta_{\bar{h}} &= \frac{54 h \bar{h}^2}{(4\pi)^2} \\
 \beta_{h_3} &= \frac{1}{(4\pi)^2} (54 h \bar{h} h_3 - 24 g^2 h + 24 g^2 h t_1 - 6 g^2 h t_1^2 - 12 h h_4) \\
 \beta_{t_1} &= \frac{36}{(4\pi)^2} (3 t_1 - 2) h \bar{h} \\
 \beta_{t_2} &= \frac{54 t_2 h \bar{h}}{(4\pi)^2} \\
 \beta_{t_3} &= \frac{1}{(4\pi)^2} (108 t_3 h \bar{h} - 72 h \bar{h} + 72 h t_2) \\
 \beta_{h_4} &= \frac{1}{(4\pi)^2} (72 h \bar{h} g^2 t_1^2 - 144 h \bar{h} g^2 t_1 - 1296 h_3 h \bar{h}^2 - 648 (h \bar{h})^2 + 360 h \bar{h} h_4 - 72 h h_5) \\
 \beta_{h_5} &= \frac{1}{(4\pi)^2} (-216 h \bar{h}^2 g^2 t_1^2 - 432 h \bar{h}^2 h_4 + 378 h \bar{h} h_5)
 \end{aligned} \tag{3.5}$$

4 Three-flavor case: renormalization and β -functions

In this section we consider the case of the NAC $U_*(1)$ gauge theory in interaction with matter in the adjoint representation of the gauge group and in the fundamental representation of a flavor $SU(3)$ group. Its action is given in (2.13). We note that in the case $h_1 = h_2, \bar{h}_1 = \bar{h}_2$, setting $\mathcal{F}^{\alpha\beta} = 0$ turns off all the interactions and we are back to the ordinary free $U(1)$ $N = 4$ SYM theory. On the other hand, the noncommutative nature of the star product allows us to construct even in the "abelian" case non-trivial interacting theories which can be studied as NAC deformations of $N = 4$ SYM. More generally, we will consider $h_1 \neq h_2, \bar{h}_1 \neq \bar{h}_2$ in order to take into account also marginal deformations.

We perform one-loop renormalization of the theory. Comparing to the case of a single chiral field, we note that the couplings are exactly of the same kind but dressed by flavor except for the extra coupling \tilde{h}_3 which in the previous case was trivially zero. Therefore, in order to evaluate divergent diagrams, it is sufficient to generalize the previous calculations to take into account non-trivial flavor factors and add possible new contributions arising from the contraction of a \tilde{h}_3 vertex with the rest. Since the \tilde{h}_3 vertex has the same structure of the vertex obtained when first order expanding the $*$ -product in the superpotential (see vertices (5f) and (5h) in (A.30)), the topologies of divergent diagrams are still the ones in figure 2, 3.

From a direct evaluation of all the contributions, for the one-loop divergent part of the effective action we find (in order to shorten the notation we define $h_{12} \equiv h_1 - h_2$ and $\bar{h}_{12} \equiv \bar{h}_1 - \bar{h}_2$)

$$\begin{aligned}
 \Gamma_{\text{div}}^{(1)} &= \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} + \int d^4x d^2\theta \Phi_i \bar{\Phi}^i [1 + h_{12} \bar{h}_{12} \mathcal{S}] \\
 &+ h_1 \int d^4x d^2\theta \Phi_1 \Phi_2 \Phi_3 - h_2 \int d^4x d^2\theta \Phi_1 \Phi_3 \Phi_2 \\
 &+ \bar{h}_1 \int d^4x d^2\bar{\theta} \bar{\Phi}^1 \bar{\Phi}^2 \bar{\Phi}^3 - \bar{h}_2 \int d^4x d^2\bar{\theta} \bar{\Phi}^1 \bar{\Phi}^3 \bar{\Phi}^2 \\
 &+ \tilde{h}_3 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \nabla_{\alpha} \Phi_1 * \nabla_{\beta} \Phi_2 * \Phi_3 \\
 &+ \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi_1 \nabla^2 \Phi_2 \nabla^2 \Phi_3 \left[h_3 + \left(12g^2 h_{12} - 6g^2 t_1 h_{12} + 3g^2 t_1^2 h_{12} + 3h_{12} h_4^{(\neq)} \right) \mathcal{S} \right] \\
 &+ i \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \partial_{\alpha}^{\dot{\alpha}} \bar{\Gamma}_{\beta\dot{\alpha}} \Phi_i \bar{\Phi}^i \left[t_1 + 2h_{12} \bar{h}_{12} (1 - t_1) \mathcal{S} \right] \\
 &+ t_2 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}^{\alpha\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}} \bar{\Phi}^1 \bar{\Phi}^2 \bar{\Phi}^3 \\
 &+ \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \Phi_i \bar{\Phi}^i \left[t_3 + 2 \left(h_{12} \bar{h}_{12} - h_{12} \bar{h}_{12} t_3 - h_{12} t_2 \right) \mathcal{S} \right] \\
 &+ \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \sum_i \nabla^2 \Phi_i \Phi_i \bar{\Phi}^i \bar{\Phi}^i \left\{ h_4^{(=)} + \left[(h_3 + \tilde{h}_3) h_{12} \bar{h}_{12}^2 - 2h_1 h_2 \bar{h}_{12}^2 - 2h_{12} \bar{h}_{12} h_4^{(\neq)} \right. \right. \\
 &\qquad \qquad \qquad \left. \left. + h_{12} h_5 - \frac{1}{2} \left(\tilde{h}_3^2 + 2(h_1 + h_2) \tilde{h}_3 \right) \bar{h}_{12}^2 \right] \mathcal{S} \right\} \\
 &+ \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \sum_{i < j} \nabla^2 \Phi_i \Phi_j \bar{\Phi}^i \bar{\Phi}^j \left\{ h_4^{(\neq)} + \left[8h_{12} \bar{h}_{12} g^2 t_1 - 4h_{12} \bar{h}_{12} g^2 t_1^2 - 8h_{12} \bar{h}_{12} h_4^{(=)} \right. \right. \\
 &+ 2(h_1^2 + h_2^2) \bar{h}_{12}^2 + 2(h_3 + \tilde{h}_3) h_{12} \bar{h}_{12}^2 + \left. \left. \left(\tilde{h}_3^2 + 2(h_1 + h_2) \tilde{h}_3 \right) \bar{h}_{12}^2 - 4h_{12} \bar{h}_{12} h_4^{(\neq)} + 4h_{12} h_5 \right] \mathcal{S} \right\} \\
 &+ \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi_i \bar{\Phi}^i \bar{\Phi}^1 \bar{\Phi}^2 \bar{\Phi}^3 \left[h_5 + \left(4h_{12} \bar{h}_{12} g^2 t_1^2 + 2h_{12} \bar{h}_{12} h_4^{(=)} \right. \right. \\
 &\qquad \qquad \qquad \left. \left. + 3h_{12} \bar{h}_{12} h_4^{(\neq)} - 6h_{12} \bar{h}_{12} h_5 \right) \mathcal{S} \right]
 \end{aligned} \tag{4.1}$$

As in the previous case the gauge sector of the theory does not receive divergent contributions. Moreover, the quadratic matter action does not receive contributions from quantum gauge fields.

Renormalization is still performed by using renormalized field functions and coupling constants as defined in (3.2). Choosing the same renormalization constants for the three (anti)chiral superfields, in minimal subtraction scheme we set

$$\begin{aligned}
 Z_i &= \bar{Z}_i = 1 - \frac{h_{12} \bar{h}_{12}}{(4\pi)^2} \frac{1}{\epsilon} \\
 Z_{h_1} &= Z_{\bar{h}_1} = Z_{h_2} = Z_{\bar{h}_2} = Z_{\tilde{h}_3} = 1 + \frac{3h_{12} \bar{h}_{12}}{2(4\pi)^2} \frac{1}{\epsilon} \\
 Z_{h_3} h_3 &= h_3 + \frac{3h_3 h_{12} \bar{h}_{12} - 24g^2 h_{12} + 12g^2 t_1 h_{12} - 6g^2 t_1^2 h_{12} - 6h_{12} h_4^{(\neq)}}{2(4\pi)^2} \frac{1}{\epsilon} \\
 Z_{t_1} t_1 &= t_1 + (3t_1 - 2) \frac{h_{12} \bar{h}_{12}}{(4\pi)^2} \frac{1}{\epsilon}
 \end{aligned} \tag{4.2}$$

$$\begin{aligned}
 Z_{t_2 t_2} &= t_2 + \frac{3h_{12}\bar{h}_{12}t_2}{2(4\pi)^2} \frac{1}{\epsilon} \\
 Z_{t_3 t_3} &= t_3 + \frac{3h_{12}\bar{h}_{12}t_3 - 2h_{12}\bar{h}_{12} + 2h_{12}t_2}{(4\pi)^2} \frac{1}{\epsilon} \\
 Z_{h_4^{(=)} h_4^{(=)}} &= h_4^{(=)} + \frac{1}{(4\pi)^2} \left[2h_{12}\bar{h}_{12}h_4^{(\neq)} - h_{12}h_5 + 2h_{12}\bar{h}_{12}h_4^{(=)} + 2h_1h_2\bar{h}_{12}^2 \right. \\
 &\quad \left. - (h_3 + \tilde{h}_3)h_{12}\bar{h}_{12}^2 + \frac{1}{2} \left(\tilde{h}_3^2 + 2(h_1+h_2)\tilde{h}_3 \right) \bar{h}_{12}^2 \right] \frac{1}{\epsilon} \\
 Z_{h_4^{(\neq)} h_4^{(\neq)}} &= h_4^{(\neq)} + \frac{1}{(4\pi)^2} \left[4h_{12}\bar{h}_{12}g^2t_1^2 - 8h_{12}\bar{h}_{12}g^2t_1 - 2(h_1^2 + h_2^2)\bar{h}_{12}^2 + 8h_{12}\bar{h}_{12}h_4^{(=)} \right. \\
 &\quad \left. + 6h_{12}\bar{h}_{12}h_4^{(\neq)} - 4h_{12}h_5 - 2(h_3 + \tilde{h}_3)h_{12}\bar{h}_{12}^2 - \left(\tilde{h}_3^2 + 2(h_1+h_2)\tilde{h}_3 \right) \bar{h}_{12}^2 \right] \frac{1}{\epsilon} \\
 Z_{h_5 h_5} &= h_5 - \frac{1}{(4\pi)^2} \left(4h_{12}\bar{h}_{12}g^2t_1^2 + 2h_{12}\bar{h}_{12}h_4^{(=)} + 3h_{12}\bar{h}_{12}h_4^{(\neq)} - \frac{17}{2}h_{12}\bar{h}_{12}h_5 \right) \frac{1}{\epsilon}
 \end{aligned}$$

Finally, applying the prescription (3.4) we find the beta-functions of the theory

$$\begin{aligned}
 \beta_g &= 0 \\
 \beta_{h_1} &= \frac{3}{(4\pi)^2} h_1 h_{12} \bar{h}_{12} & \beta_{\bar{h}_1} &= \frac{3}{(4\pi)^2} \bar{h}_1 h_{12} \bar{h}_{12} \\
 \beta_{h_2} &= -\frac{3}{(4\pi)^2} h_2 h_{12} \bar{h}_{12} & \beta_{\bar{h}_2} &= -\frac{3}{(4\pi)^2} \bar{h}_2 h_{12} \bar{h}_{12} \\
 \beta_{\tilde{h}_3} &= \frac{3}{(4\pi)^2} h_{12} \bar{h}_{12} \tilde{h}_3 \\
 \beta_{h_3} &= \frac{1}{(4\pi)^2} \left(3h_{12}\bar{h}_{12}h_3 - 24g^2h_{12} + 12g^2t_1h_{12} - 6g^2t_1^2h_{12} - 6h_{12}h_4^{(\neq)} \right) \\
 \beta_{t_1} &= \frac{2}{(4\pi)^2} (3t_1 - 2)h_{12}\bar{h}_{12} \\
 \beta_{t_2} &= \frac{3}{(4\pi)^2} h_{12}\bar{h}_{12}t_2 \\
 \beta_{t_3} &= \frac{1}{(4\pi)^2} (6h_{12}\bar{h}_{12}t_3 - 4h_{12}\bar{h}_{12} + 4h_{12}t_2) \\
 \beta_{h_4^{(=)}} &= \frac{1}{(4\pi)^2} \left[4h_{12}\bar{h}_{12}h_4^{(\neq)} + 4h_{12}\bar{h}_{12}h_4^{(=)} - 2h_{12}h_5 + 4h_1h_2\bar{h}_{12}^2 \right. \\
 &\quad \left. - 2(h_3 + \tilde{h}_3)h_{12}\bar{h}_{12}^2 + \left(\tilde{h}_3^2 + 2(h_1+h_2)\tilde{h}_3 \right) \bar{h}_{12}^2 \right] \\
 \beta_{h_4^{(\neq)}} &= \frac{1}{(4\pi)^2} \left[8h_{12}\bar{h}_{12}g^2t_1^2 - 16h_{12}\bar{h}_{12}g^2t_1 + 16h_{12}\bar{h}_{12}h_4^{(=)} + 12h_{12}\bar{h}_{12}h_4^{(\neq)} - 8h_{12}h_5 \right. \\
 &\quad \left. - 4(h_1^2 + h_2^2)\bar{h}_{12}^2 - 4(h_3 + \tilde{h}_3)h_{12}\bar{h}_{12}^2 - 2 \left(\tilde{h}_3^2 + 2(h_1+h_2)\tilde{h}_3 \right) \bar{h}_{12}^2 \right] \\
 \beta_{h_5} &= -\frac{1}{(4\pi)^2} \left(8h_{12}\bar{h}_{12}g^2t_1^2 + 4h_{12}\bar{h}_{12}h_4^{(=)} + 6h_{12}\bar{h}_{12}h_4^{(\neq)} - 17h_{12}\bar{h}_{12}h_5 \right)
 \end{aligned}$$

5 Finiteness, fixed points and IR stability

We now discuss the previous results for different choices of the chiral couplings. We recall that we are working with euclidean theories which are not subject to hermitian conjugation constraints. In particular, Φ and $\bar{\Phi}$ are independent real superfields as well as the corresponding couplings h and \bar{h} .

We first consider the case of the theory with a single chiral superfield. Referring to the results (3.1) we note that all the divergences are proportional to (powers of) the superpotential coupling h . Therefore, setting $h = 0$ the theory turns out to be one-loop finite and we have no need to add all possible couplings in order to get a renormalizable theory. Precisely, the following action

$$S = \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} + \int d^4x d^4\theta \Phi * \bar{\Phi} + \bar{h} \int d^4x d^2\bar{\theta} \bar{\Phi}^3 \quad (5.1)$$

is perfectly consistent at quantum level and one-loop finite.

Conversely, if we set $\bar{h} = 0$ while keeping the chiral superpotential on we have few divergent contributions surviving in (3.1). Making the minimal choice of setting to zero all the extra couplings which do not get renormalized we find that the following action is one-loop renormalizable

$$S = \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} + \int d^4x d^4\theta \Phi * \bar{\Phi} + h \int d^4x d^2\theta \Phi_*^3 + h_3 \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi * \nabla^2 \Phi * \nabla^2 \Phi \quad (5.2)$$

but not finite. This result is consistent with what has been found [7–9] for the NAC ungauged Wess-Zumino model.

The fact that the theory is finite when we turn off the superpotential in the chiral sector while tolerating a superpotential for antichirals but not viceversa is a manifestation of the asymmetry between the chiral and the antichiral sectors induced by the star product.

We now discuss the spectrum of fixed points for the most general case where all the couplings are turned on. As already seen, the theory is one-loop finite when we set $h = 0$, independently of the value of the other couplings. Therefore, $h = 0$ defines an eight dimensional hypersurface of fixed points.

However, $h = 0$ does not exhaust the spectrum of fixed points. In fact, by a quick look at the beta functions in (3.5) we can easily see that taking $h \neq 0$ there is another hypersurface of fixed points given by

$$\begin{aligned} \bar{h} = h_5 = t_2 = 0 \\ 2h_4 + g^2(t_1 - 2)^2 = 0 \end{aligned} \quad (5.3)$$

In any case, from the requirement for $\beta_h, \beta_{\bar{h}}$ to vanish we are forced to set either h or \bar{h} equal to zero. This is due to the fact that, despite the non-trivial gauge/matter interaction, the matter quadratic term does not get corrections from gauge quantum fields. As a consequence, we do not have non-trivial $h(g), \bar{h}(g)$ functions which describe marginal flows as it happens in ordinary non-abelian SYM theories.

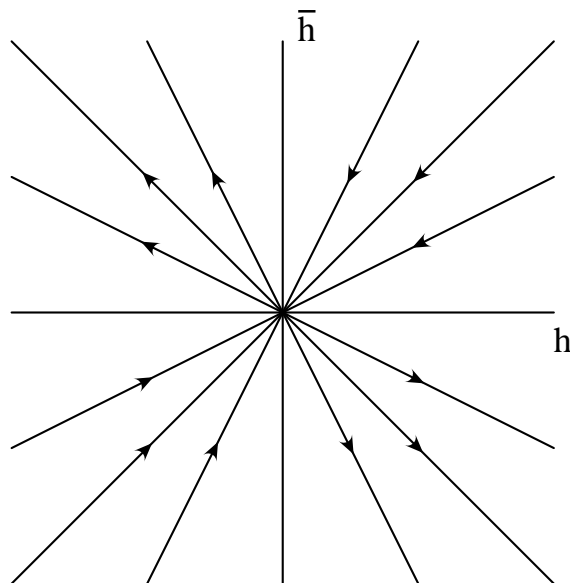


Figure 4. Renormalization group trajectories near the $h = \bar{h} = 0$ fixed point. Arrows indicate the IR flows.

We study the stability of fixed points and compare the present situation with the corresponding anticommutative case, that is an ordinary abelian SYM theory perturbed by a cubic superpotential

$$h \int d^4x d^2\theta \Phi^3 + \bar{h} \int d^4x d^2\bar{\theta} \bar{\Phi}^3 \tag{5.4}$$

where hermiticity requires \bar{h} to be the complex conjugate of h .

In the ordinary case the theory is simply a free gauge theory plus a massless Wess-Zumino model. The corresponding one-loop β -functions go like $\beta_h \sim |h|^2 h$ and $\beta_{\bar{h}} \sim |h|^2 \bar{h}$. Therefore, the only fixed point of the theory is $h = \bar{h} = 0$. The RG trajectories are drawn in figure 4 where only the first and third quadrants have to be considered ($h\bar{h} = |h|^2 \geq 0$). Therefore, the origin corresponds to an IR stable fixed point.

We now consider the NAC case described by the general action (2.12). The great number of coupling constants forbids plotting global RG trajectories; however, we can study the IR behavior of the theory on lower dimensional hypersurfaces by temporarily keeping a certain number of couplings fixed. First of all, since $\beta_g = 0$ we can sit on hypersurfaces $g = \bar{g}$ where \bar{g} is a small constant. Moreover, we can identify the flows associated to β_h and $\beta_{\bar{h}}$ as a closed subset of equations.

The main difference compared to the ordinary case is that now h and \bar{h} are two *real* independent couplings. This has two consequences: 1) The spectrum of fixed points is now given by the two lines $h = 0$ and $\bar{h} = 0$; 2) Since the product $h\bar{h}$ can be either positive or negative we need extend the study of RG trajectories to the whole (h, \bar{h}) plane.

The configuration of RG trajectories is given in figure 4 where arrows indicate the IR flow. It is easy to see that the two axes $h = 0$ and $\bar{h} = 0$ are lines of unstable fixed points.

In particular, we see that in this case the origin is neither an infrared nor an ultraviolet attractor. This is in contrast with the ordinary case where, as discussed above, the origin is an IR stable fixed point. The different behavior of the two theories can be traced back to the different hermiticity conditions which constrain the (anti)chiral coupling constants.

Although the failure of the origin to be an IR attractor is conclusive, we can restrict the couplings to have the same sign (then studying the flows in the first and third quadrants) and investigate whether we can identify a region in the parameter space for which the origin is an infrared attractor.

The $(h, \bar{h}) = (0, 0)$ fixed point spans a seven dimensional hypersurface of fixed points corresponding to all possible values of the other couplings. We study RG trajectories on this hypersurface by linearizing in the rest of the couplings.

The system of linearized equations we consider is $\mu dh_i/d\mu = \beta_{h_i}, i = 3, 4, 5$, while the remaining equations decouple and have stability matrix with positive eigenvalues. Keeping (h, \bar{h}) slightly away from the fixed point, the eigenvalues of the stability matrix for the subset (h_3, h_4, h_5) are approximatively

$$\rho_1 = -1.608 h \bar{h} \quad \rho_2 = 232.788 h \bar{h} \quad \rho_3 = 560.82 h \bar{h} \quad (5.5)$$

We see that the matrix vanishes at the fixed point but, as soon as we move away from the fixed point, there is at least one negative eigenvalue in any quadrant of the (h, \bar{h}) -plane. The corresponding eigenvector represents an instability direction and leads to the conclusion that the origin is never an IR attractor whatever the range for (h, \bar{h}) is.

We now consider the more interesting case of three flavors. As already stressed, the theory (2.13) describes a NAC generalization of the abelian $N = 4$ SYM theory and theories obtained from it by adding marginal deformations.

We remind that the ordinary abelian $N = 4$ SYM theory is a free theory, then necessarily finite. Marginal deformations can be added of the form (we write them in a form which can be easily generalized to the NAC case)

$$\int d^4x d^2\theta (h_1 \Phi_1 \Phi_2 \Phi_3 - h_2 \Phi_1 \Phi_3 \Phi_2) + \int d^4x d^2\bar{\theta} (\bar{h}_1 \bar{\Phi}^1 \bar{\Phi}^2 \bar{\Phi}^3 - \bar{h}_2 \bar{\Phi}^1 \bar{\Phi}^3 \bar{\Phi}^2) \quad (5.6)$$

which break supersymmetry down to $N = 1$. In our notation $N = 4$ supersymmetry is recovered for $h_1 = h_2$ ($\bar{h}_1 = \bar{h}_2$ are the hermitian conjugates). The deformed theory is no longer finite since a divergent self-energy contribution to the (anti)chirals appears at one-loop, proportional to $h_{12} \bar{h}_{12}$. It is easy to see that the free $N = 4$ theory is a stable IR fixed point.

We now study what happens in the NAC case. Looking at the results (4.1) the first important observation is that the gauge beta-function is identically zero and all the other divergences are proportional to powers of h_{12} and \bar{h}_{12} . Therefore, setting $h_1 = h_2$ and $\bar{h}_1 = \bar{h}_2$ kills all the divergences and the theory is one-loop finite. It follows that, at least at one-loop, the NAC deformation does not affect the finiteness properties of the $N = 4$ SYM theory.

It is not difficult to provide general arguments for extending this analysis to all loops. First of all, the gauge sector cannot receive loop corrections at any perturbative order. In

fact, for dimensional and symmetry reasons [18] in the $U_*(1)$ case the only local background structure which might be produced is the quadratic term $\int \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}$ with no powers of the NAC parameter in front. As already discussed, any loop diagram that we can draw contributing to the gauge sector is proportional to powers of $\mathcal{F}^{\alpha\beta}$ and then necessarily finite.

In the mixed and matter sectors, the constraints on the maximal power of $\mathcal{F}^{\alpha\beta}$ that we can have in divergent diagrams imply that at least one chiral or one antichiral vertex from the superpotential needs be present at order zero in the NAC parameter, therefore carrying a coupling h_{12} or \bar{h}_{12} . Then, it is a matter of fact that in the case of equal couplings all contributions vanish.

Therefore, on general grounds we conclude that the $U_*(1)$ deformation of the abelian $N = 4$ SYM theory is all loop finite.

Exactly marginal deformations are obtained by adding marginal operators to the action which do not affect the vanishing of the beta-functions. In our case, taking $h_1 \neq h_2$ and/or $\bar{h}_1 \neq \bar{h}_2$ means adding marginal operators. However, not all of them turn out to be exactly marginal, at least at one-loop. In fact, in order to have vanishing beta-functions away from the symmetric point $h_{12} = \bar{h}_{12} = 0$ we need require either

$$h_1 = h_2 \quad (\tilde{h}_3 + 2h_1) = 0 \quad (5.7)$$

or

$$\bar{h}_1 = \bar{h}_2 \quad t_2 = h_5 = 0 \quad g^2(t_1^2 - 2t_1 + 4) + h_4^{(\neq)} = 0 \quad (5.8)$$

In order to study the stability of the fixed points we can perform an analysis similar to the previous one with the obvious substitutions $h \rightarrow h_{12}$ and $\bar{h} \rightarrow \bar{h}_{12}$ plus the additional couplings which were not present in the one-flavor case.

The flow equations for h_{12} and \bar{h}_{12} still decouple from the rest of the system and we can first study the IR behavior of the theory restricted to the (h_{12}, \bar{h}_{12}) plane. With the suitable substitutions figure 4 is still valid and provides two lines $h_{12} = 0$ and $\bar{h}_{12} = 0$ of unstable fixed points.

Restricting the range of (h_{12}, \bar{h}_{12}) within the first and third quadrants and neglecting t_2 which has a trivial β -function, we are left with a system of seven equations whose stability matrix can be studied in a neighborhood of the origin. The corresponding eigenvalues are approximatively

$$\begin{aligned} \rho_1 &= 3 h_{12} \bar{h}_{12} & \rho_{2,3} &= 6 h_{12} \bar{h}_{12} & \rho_4 &= -0.626 h_{12} \bar{h}_{12} \\ \rho_5 &= 3.936 h_{12} \bar{h}_{12} & \rho_6 &= 11.674 h_{12} \bar{h}_{12} & \rho_7 &= 25.017 h_{12} \bar{h}_{12} \end{aligned} \quad (5.9)$$

Again, the appearance of at least one negative eigenvalue for any choice of the couplings leads to the conclusion that the $N = 4$ theory is not an IR attractor. This result is similar to what happens in the ordinary non-abelian SYM theories with gauge group $SU(N \geq 3)$ [24], even if the two theories are not directly mappable one onto the other.

6 Conclusions

Deforming half of the Grassmannian part of the superspace could have bad consequences on the quantum behavior of field theories defined on it. In fact, due to the partial breaking

of supersymmetry the non-trivial cancellation between bosonic and fermionic divergences is not guaranteed anymore and we can expect a worsening in the UV behavior of the theories. Equivalently, the deformation introduces a “bad” dimensionful parameter which might induce the appearance of dimensionful momentum integrals with positive degree of superficial divergence. However, for all models investigated so far, a careful analysis has revealed that consistent completions of NAC deformations of ordinary theories can always be found for which renormalizability is preserved thanks to some global symmetries inherited from the parent theory.

In this paper we have continued on this line of investigation by performing the one-loop renormalization of $U_*(1)$ SYM theories with matter in the adjoint representation of the gauge group, motivated by the idea of finding NAC generalizations of ordinary SYM with extended supersymmetry. In general, the actions are not simply obtained from the ordinary ones by deforming the products, but contain suitable completions given in terms of all classical marginal operators which respect a given set of global symmetries.

We have first considered a SYM theory with a single chiral field self-interacting through a cubic superpotential. Then, we have extended our analysis to the case of three matter fields interacting through a cubic superpotential which depends on four coupling constants, $h_1, h_2, \bar{h}_1, \bar{h}_2$. For $h_1 = h_2$ and $\bar{h}_1 = \bar{h}_2$ the classical action exhibits a global $SU(3)$ symmetry and can be interpreted as a NAC generalization of the ordinary $N = 4$ SYM theory. More generally, for $h_1 \neq h_2$ and/or $\bar{h}_1 \neq \bar{h}_2$ it looks like the NAC generalization of marginal deformations of $N = 4$ SYM.

Since in the ordinary case $N = 4$ SYM is finite, one of the questions we have addressed is whether finiteness survives in the NAC case. We note that, while in the ordinary $U(1)$ case finiteness is a trivial statement, being the theory free, its NAC generalization is highly interacting and the question becomes interesting. We have found that at one-loop the theory with $h_1 = h_2, \bar{h}_1 = \bar{h}_2$ is indeed *finite*. Moreover, based on general arguments we have provided a proof for the all-loop finiteness of the theory.

More generally, we have considered theories in the presence of marginal deformations. In this case UV divergences arise which in general set the theory away from a fixed point. In the parameter space we have studied the spectrum of fixed points and the renormalization group flows. We have found that, while in the ordinary $N = 4$ case $h_1 = h_2, \bar{h}_1 = \bar{h}_2$ is an IR stable fixed point (free theory), in our case nonanticommutativity makes all the fixed points unstable. This is due to the fact that in the presence of extra marginal operators proportional to $\mathcal{F}^{\alpha\beta}$, the parameter space gets enlarged and new lines of instability are allowed. Even if our analysis is based on one-loop calculations, we have already enough information for drawing qualitative conclusions on the effects that this kind of geometrical deformations have on the RG flows: NAC theories resemble the non-abelian $SU(N \geq 3)$ ordinary theories for which $N = 4$ SYM is neither an IR nor an UV attractor.

We focused only on massless theories but it is easy to convince that the addition of a mass term should not change the main features of the theories.

In order to simplify the analysis, we considered the $U_*(1)$ case. From the point of view of studying how renormalization works these theories are not too trivial. In fact, as already stressed, they are highly interacting. Therefore, the results obtained on the finiteness in a subspace of the parameter space and, more generally, on the role of nonanticommutativity on their UV and IR behavior are actually not *a priori* expected.

However, considering this example we have lost the non-trivial coupling between non-abelian and abelian superfields which is a peculiar feature of the NAC gauge theories. It would be then very interesting to consider the non-trivial $SU(\mathcal{N}) \otimes U(1)$ case and investigate whether the obtained results survive. In particular, it would be interesting to address the question of finiteness. In fact, we expect that at one-loop the gauge sector would not receive divergent corrections since matter loops would cancel ghost loops, still giving $\beta_g^{(1)} = 0$. In the matter sector new contributions proportional to g^2 would arise for the two and higher point functions. Therefore, as in the ordinary non abelian cases, we expect non-trivial surfaces of fixed points of the form $h_{12} = h_{12}(g), \bar{h}_{12} = \bar{h}_{12}(g)$. The non-trivial question is whether this is only a one-loop effect or it would arise as an actual feature of the whole quantum lagrangian.

From a stringy point of view, our results are a further step towards a better understanding of the dynamics of D3-branes in the presence of non-vanishing RR forms and provide few hints for constructing gravity duals.

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A Background field method and Feynman rules

In this appendix we collect all one-loop Feynman rules obtained from the actions (2.12), (2.13) by applying the generalized background field method developed in [11] for NAC super Yang-Mills theories with chiral matter in a *real* representation of the gauge group.

Gauge sector. We work in gauge antichiral representation [21] for covariant derivatives and perform the quantum-background splitting according to

$$\nabla_\alpha = \nabla_\alpha = D_\alpha, \quad \bar{\nabla}_{\dot{\alpha}} = e_*^V * \bar{\nabla}_{\dot{\alpha}} * e_*^{-V} = e_*^V * e_*^U * \bar{D}_{\dot{\alpha}} e_*^{-U} * e_*^{-V} \quad (A.1)$$

The derivatives transform covariantly with respect to quantum transformations

$$\begin{aligned} e_*^V &\rightarrow e_*^{i\bar{\Lambda}} * e_*^V * e_*^{-i\Lambda}, & e_*^U &\rightarrow e_*^U \\ \nabla_A &\rightarrow e_*^{i\bar{\Lambda}} * \nabla_A * e_*^{-i\bar{\Lambda}}, & \nabla_A &\rightarrow \nabla_A \end{aligned} \quad (A.2)$$

with background covariantly (anti)chiral parameters, $\nabla_\alpha \bar{\Lambda} = \bar{\nabla}_{\dot{\alpha}} \Lambda = 0$, and background transformations

$$\begin{aligned} e_*^V &\rightarrow e_*^{i\bar{\lambda}} * e_*^V * e_*^{-i\bar{\lambda}}, & e_*^U &\rightarrow e_*^{i\bar{\lambda}} * e_*^U * e_*^{-i\lambda} \\ \nabla_A &\rightarrow e_*^{i\bar{\lambda}} * \nabla_A * e_*^{-i\bar{\lambda}}, & \nabla_A &\rightarrow e_*^{i\bar{\lambda}} * \nabla_A * e_*^{-i\bar{\lambda}} \end{aligned} \quad (A.3)$$

with ordinary (anti)chiral parameters $\bar{D}_{\dot{\alpha}} \lambda = D_\alpha \bar{\lambda} = 0$.

The classical action

$$S_{\text{gauge}} = \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \bar{W}^{\dot{\alpha}} W_{\dot{\alpha}} \quad (\text{A.4})$$

for the gauge field strength defined in eq. (2.10) is invariant under gauge transformations (A.2) and (A.3). Background field quantization consists in performing gauge-fixing which explicitly breaks the (A.2) gauge invariance while preserving manifest invariance of the effective action and correlation functions under (A.3). Choosing as in the ordinary case the gauge-fixing functions as $f = \bar{\nabla}^2 * V$, $\bar{f} = \nabla^2 * V$ the resulting gauge-fixed action has exactly the same structure as in the ordinary case [21] with products promoted to star products [11]. In Feynman gauge it reads

$$\begin{aligned} S_{\text{gauge}} + S_{\text{GF}} + S_{\text{gh}} = & \\ & -\frac{1}{2g^2} \int d^4x d^4\theta \left[e_*^V * \bar{\nabla}^{\dot{\alpha}} * e_*^{-V} * D^2(e_*^V * \bar{\nabla}_{\dot{\alpha}} * e_*^{-V}) + V * (\bar{\nabla}^2 D^2 + D^2 \bar{\nabla}^2) * V \right] \\ & + \int d^4x d^4\theta \left[\bar{c}' c - c' \bar{c} + \dots + \bar{b} b \right] \end{aligned} \quad (\text{A.5})$$

where ghosts are background covariantly (anti)chiral superfields and dots stand for higher order interaction terms.

What we have reviewed so far holds for any NAC gauge theory, independently of the choice of the gauge group. Now, we focus on the case we are interested in, that is $U_*(1)$ and determine the Feynman rules.

Working out the quadratic part of the action from (A.5) we find

$$S + S_{\text{GF}} \rightarrow -\frac{1}{2g^2} \int d^4x d^4\theta V * \hat{\square} * V \quad (\text{A.6})$$

where we have defined

$$\hat{\square} = \square_{\text{cov}} - i\widetilde{\mathbf{W}}^{\alpha} * \nabla_{\alpha} - i\bar{\mathbf{W}}^{\dot{\alpha}} * \bar{\nabla}_{\dot{\alpha}}, \quad \square_{\text{cov}} = \frac{1}{2} \bar{\nabla}^{\alpha\dot{\alpha}} * \bar{\nabla}_{\alpha\dot{\alpha}} \quad (\text{A.7})$$

We find convenient to rescale the gauge field as

$$V \rightarrow gV \quad (\text{A.8})$$

Therefore, from the rescaled action we determine the covariant propagator

$$\langle V(z)V(z') \rangle = \frac{1}{\hat{\square}} \delta^{(8)}(z - z') \quad (\text{A.9})$$

where $z \equiv (x^{\alpha\dot{\alpha}}, \theta^{\alpha}, \bar{\theta}^{\dot{\alpha}})$.

Expanding this expression in powers of the background fields it turns out that the covariant propagator contains an infinite number of background-quantum interaction vertices. Precisely, we write

$$\frac{1}{\hat{\square}} \simeq \frac{1}{\square_{\text{cov}}} + \frac{1}{\square_{\text{cov}}} * \left(i\widetilde{\mathbf{W}}^{\alpha} * \nabla_{\alpha} + i\bar{\mathbf{W}}^{\dot{\alpha}} * \bar{\nabla}_{\dot{\alpha}} \right) * \frac{1}{\square_{\text{cov}}} + \dots \quad (\text{A.10})$$

and further expand $1/\square_{\text{cov}}$. Since by direct inspection one can easily realize that terms proportional to $\widetilde{\mathbf{W}}^\alpha$ and $\overline{\mathbf{W}}^{\dot{\alpha}}$ never enter one-loop divergent diagrams, we approximate

$$\frac{1}{\hat{\square}} \simeq \frac{1}{\square_{\text{cov}}} \quad (\text{A.11})$$

and study in detail its expansion.

On a generic superfield in the adjoint representation of the gauge group we have

$$\begin{aligned} \square_{\text{cov}} * \phi &\equiv \frac{1}{2} [\overline{\nabla}^{\alpha\dot{\alpha}}, [\overline{\nabla}_{\alpha\dot{\alpha}}, \phi]_*]_* \\ &= \square\phi - i[\overline{\Gamma}^{\alpha\dot{\alpha}}, \partial_{\alpha\dot{\alpha}}\phi]_* - \frac{i}{2} [(\partial^{\alpha\dot{\alpha}}\overline{\Gamma}_{\alpha\dot{\alpha}}), \phi]_* - \frac{1}{2} [\overline{\Gamma}^{\alpha\dot{\alpha}}, [\overline{\Gamma}_{\alpha\dot{\alpha}}, \phi]_*]_* \end{aligned} \quad (\text{A.12})$$

where $\square = \frac{1}{2}\partial^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$ is the ordinary scalar kinetic term.

Expanding the $*$ -products and neglecting terms which never enter our calculations we find

$$\square_{\text{cov}} = \square + 2i\mathcal{F}^{\alpha\beta}(\partial_\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})\partial_\beta\partial_{\gamma\dot{\gamma}} - \mathcal{F}^2(\partial^\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})(\partial^2\overline{\Gamma}_{\gamma\dot{\gamma}})\partial_\alpha + \mathcal{F}^2(\partial^\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})(\partial_\alpha\overline{\Gamma}_{\gamma\dot{\gamma}})\partial^2 + \dots \quad (\text{A.13})$$

Inverting this expression we finally have

$$\begin{aligned} \frac{1}{\square_{\text{cov}}} &= \frac{1}{\square} \\ &- \frac{1}{\square} 2i\mathcal{F}^{\alpha\beta}(\partial_\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})\partial_\beta\partial_{\gamma\dot{\gamma}}\frac{1}{\square} - \frac{1}{\square} 4\mathcal{F}^{\alpha\beta}(\partial_\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})\partial_\beta\partial_{\gamma\dot{\gamma}}\frac{1}{\square}\mathcal{F}^{\eta\rho}(\partial_\eta\overline{\Gamma}^{\sigma\dot{\sigma}})\partial_\rho\partial_{\sigma\dot{\sigma}}\frac{1}{\square} \\ &+ \frac{1}{\square}\mathcal{F}^2(\partial^\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})(\partial^2\overline{\Gamma}_{\gamma\dot{\gamma}})\partial_\alpha\frac{1}{\square} - \frac{1}{\square}\mathcal{F}^2(\partial^\alpha\overline{\Gamma}^{\gamma\dot{\gamma}})(\partial_\alpha\overline{\Gamma}_{\gamma\dot{\gamma}})\partial^2\frac{1}{\square} + \dots \end{aligned} \quad (\text{A.14})$$

Here we recognize the ordinary bare propagator $1/\square$ plus a number of gauge interaction vertices. We note that all the interactions are proportional to the NAC parameter, as a peculiar feature of the $U_*(1)$ theory.

Matter sector. In background field method we define *full* (anti)chiral superfields in the adjoint representation of the gauge group as

$$\overline{\Phi} = \overline{\Phi}, \quad \Phi = e_*^V * \Phi * e_*^{-V} = e_*^V * (e_*^U * \phi * e_*^{-U}) * e_*^{-V} \quad (\text{A.15})$$

where $\overline{\Phi} \equiv e_*^U * \phi * e_*^{-U}$ and $\overline{\Phi}$ are *background* covariantly (anti)chirals.

Under both quantum (A.2) and background (A.3) transformations the full (anti)chiral superfields transform covariantly with parameters $\overline{\Lambda}$ and $\overline{\lambda}$, respectively.

Under quantum transformations background covariantly (anti)chiral fields transform as $\Phi' = e_*^{i\Lambda} * \Phi * e_*^{-i\Lambda}$, $\overline{\Phi}' = e_*^{i\overline{\Lambda}} * \overline{\Phi} * e_*^{-i\overline{\Lambda}}$. Under background transformations they both transform covariantly with parameter $\overline{\lambda}$, $\Phi' = e_*^{i\overline{\lambda}} * \Phi * e_*^{-i\overline{\lambda}}$, $\overline{\Phi}' = e_*^{i\overline{\lambda}} * \overline{\Phi} * e_*^{-i\overline{\lambda}}$.

Focusing the discussion on the $U_*(1)$ gauge group we now derive propagators and interaction vertices for matter in the actions (2.12), (2.13) where we have performed the rescaling (A.8). Since one-loop divergent contributions are at most quadratic in the NAC parameter, we list only Feynman rules entering these kinds of terms.

We split the actions (2.12), (2.13) according to

$$S \equiv S_{\text{gauge}} + S_{\text{matter}} = S_{\text{gauge}} + \int d^4x d^4\theta \bar{\Phi} * \Phi + S_{\text{int}} \quad (\text{A.16})$$

where S_{gauge} is given in (A.5) and S_{int} is the rest of the matter actions in (2.12), (2.13) subtracted by the quadratic part.

We concentrate on S_{matter} . Its quantization proceeds as usual. We first expand the full covariant quadratic action in terms of background covariantly (anti)chiral fields (see (A.15))

$$\begin{aligned} & \int d^4x d^4\theta \bar{\Phi} * e^{gV} * \Phi * e^{-gV} \\ &= \int d^4x d^4\theta \left\{ \bar{\Phi} \Phi + g \bar{\Phi} [V, \Phi]_* + \frac{g^2}{2} \bar{\Phi} [V, [V, \Phi]_*]_* + \dots \right\} \end{aligned} \quad (\text{A.17})$$

The first term in this expansion is the kinetic term for background covariantly (anti)chiral fields. In particular, ghosts fall in this category so the same procedure can be applied to the action (2.17), as well. The remaining terms give rise to ordinary interactions with the quantum field V .

We perform the quantum-background splitting

$$\Phi \rightarrow \Phi + \Phi_q, \quad \bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_q \quad (\text{A.18})$$

which allows to rewrite

$$S_{\text{matter}} = \int d^4x d^4\theta \bar{\Phi}_q \Phi_q + S'_{\text{int}} \quad (\text{A.19})$$

where S'_{int} collects all the interaction vertices coming from S_{int} after the splitting (A.18) plus the extra interactions from (A.17).

Adding source terms

$$\int d^4x d^2\theta j \Phi_q + \int d^4x d^2\bar{\theta} \bar{\Phi}_q \bar{j} \quad (\text{A.20})$$

and performing the gaussian integral in $\Phi_q, \bar{\Phi}_q$, the quantum partition function reads

$$\mathbf{Z}[j, \bar{j}] = \Delta_* * e^{S'_{\text{int}}(\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{j}})} \exp \left[-\frac{1}{2} \int d^4x d^4\theta \left(j * \frac{1}{\square_-} * \bar{j} + \bar{j} * \frac{1}{\square_+} * j \right) \right] \quad (\text{A.21})$$

where we have defined

$$\begin{aligned} \square_+ &= \square_{\text{cov}} - i \widetilde{W}^\alpha * \nabla_\alpha - \frac{i}{2} (\nabla^\alpha * \widetilde{W}_\alpha) \\ \square_- &= \square_{\text{cov}} - i \overline{W}^{\dot{\alpha}} * \overline{\nabla}_{\dot{\alpha}} - \frac{i}{2} (\overline{\nabla}^{\dot{\alpha}} * \overline{W}_{\dot{\alpha}}) \end{aligned} \quad (\text{A.22})$$

and Δ_* is the functional determinant

$$\Delta_* = \int \mathcal{D}\Phi_q \mathcal{D}\bar{\Phi}_q \exp \int d^4x d^4\theta \bar{\Phi}_q \Phi_q \quad (\text{A.23})$$

From the generating functional (A.21) we have two types of perturbative contributions, one from the expansion of Δ_* and one from the expansion of $\exp(S'_{\text{int}})$.

As explained in [11, 21], Δ_* provides an additional, one-loop contribution to the gauge effective action coming from matter/ghost loops. The corresponding Feynman rules can be worked out by applying the “doubling trick” procedure [11, 21]. As a result, one-loop Feynman rules are obtained which can be formally read from the following effective action

$$\int d^4x d^4\theta \operatorname{Tr} \left\{ \bar{\xi} \square \xi + \frac{1}{2} \left[\bar{\xi} D^2 (\bar{\nabla}^2 - \bar{D}^2) \xi + \bar{\xi} (\square_- - \square) \xi \right] \right\} \quad (\text{A.24})$$

where ξ , $\bar{\xi}$ are *unconstrained* quantum fields with ordinary scalar propagator

$$\langle \xi(z) \bar{\xi}(z') \rangle = -\frac{1}{\square} \delta^{(8)}(z - z') \quad (\text{A.25})$$

and the first vertex must appear once, and only once, in a one-loop diagram.

The second type of contributions come from the expansion of $\exp(S'_{\text{int}})$ in (A.21). The covariant matter propagators in this case are

$$\begin{aligned} \langle \Phi(z) \bar{\Phi}(z') \rangle &= -\frac{1}{\square_-} \delta^{(8)}(z - z') \\ \langle \bar{\Phi}(z) \Phi(z') \rangle &= -\frac{1}{\square_+} \delta^{(8)}(z - z') \end{aligned} \quad (\text{A.26})$$

which can be expanded according to

$$\begin{aligned} \frac{1}{\square_-} &\simeq \frac{1}{\square_{\text{cov}}} + \frac{1}{\square_{\text{cov}}} * \left(i \bar{W}^{\dot{\alpha}} * \bar{\nabla}_{\dot{\alpha}} + \frac{i}{2} (\bar{\nabla}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}}) \right) * \frac{1}{\square_{\text{cov}}} + \dots \\ \frac{1}{\square_+} &\simeq \frac{1}{\square_{\text{cov}}} + \frac{1}{\square_{\text{cov}}} * \left(i \widetilde{W}^{\alpha} * \nabla_{\alpha} + \frac{i}{2} (\nabla^{\alpha} * \widetilde{W}_{\alpha}) \right) * \frac{1}{\square_{\text{cov}}} + \dots \end{aligned} \quad (\text{A.27})$$

and contain an infinite number of interaction vertices between background gauge fields and quantum matter fields. As explained in the text, at one-loop divergent contributions arise only from the $\frac{1}{\square_{\text{cov}}}$ part of the propagators. Therefore, we will set

$$\frac{1}{\square_{\pm}} \simeq \frac{1}{\square_{\text{cov}}} \quad (\text{A.28})$$

and further expand it as done in (A.14).

Interaction vertices are obtained by working out the actual expression of S'_{int} after the background-quantum splitting (A.18). We list only the ones which effectively enter the evaluation of divergences. To keep the discussion more general we consider the three-flavor case. The one-flavor vertices are then easily obtained by dropping flavor indices and neglecting terms that, without flavors, vanish for symmetry reasons.

We begin by considering the contributions (A.17) coming from the quadratic action. The only contributing vertex is (5a) in figure 5 where V is quantum and Φ or $\bar{\Phi}$ are background. We then consider the t_1, t_2, t_3 interaction terms in (2.13). Because of the presence of a $\bar{\theta}^2$ the $*$ -products are actually ordinary products. The quantization proceeds by performing the splitting (A.18) on the (anti)chirals and expanding the connections and

the field strength as follows

$$\begin{aligned}
 \bar{\Gamma}_{\alpha\dot{\alpha}} &\rightarrow -\nabla_{\alpha}e^{-V}\bar{\nabla}_{\dot{\alpha}}e^V \rightarrow \bar{\Gamma}_{\alpha\dot{\alpha}} - \nabla_{\alpha}[\bar{\nabla}_{\dot{\alpha}}, V]_* + \frac{1}{2}\nabla_{\alpha}[[\bar{\nabla}_{\dot{\alpha}}, V]_*, V]_* \\
 \partial_{\beta\dot{\alpha}}\bar{\Gamma}_{\alpha}^{\dot{\alpha}} &\rightarrow \partial_{\beta\dot{\alpha}}\bar{\Gamma}_{\alpha}^{\dot{\alpha}} - \partial_{\beta\dot{\alpha}}\nabla_{\alpha}[\bar{\nabla}_{\dot{\alpha}}, V]_* \\
 \bar{W}_{\dot{\alpha}} &\rightarrow -i\nabla^2e^{-V}\bar{\nabla}_{\dot{\alpha}}e^V \rightarrow \bar{W}_{\dot{\alpha}} - i\nabla^2[\bar{\nabla}_{\dot{\alpha}}, V]_* + \frac{i}{2}\nabla^2[[\bar{\nabla}_{\dot{\alpha}}, V]_*, V]_* \quad (\text{A.29})
 \end{aligned}$$

Collecting only the contributions which may contribute at one-loop we obtain vertices (5b, 5d) where gauge is only background and vertex (5c) where Φ or $\bar{\Phi}$ are background. We note that they all exhibit a gauge-invariant background dependence. We then turn to the pure matter interaction terms. By splitting (anti)chiral superfields we find vertices (5f – 5m).

Collecting all the results, the explicit expressions for the vertices are

$$\begin{aligned}
 (5a) \quad &-2ig\bar{\theta}^{\dot{\alpha}}\mathcal{F}^{\alpha\beta}V(\partial_{\alpha}\Phi_i)\partial_{\beta\dot{\alpha}}\bar{\Phi}^i \\
 (5b) \quad &it_1\bar{\theta}^2\mathcal{F}^{\alpha\beta}(\partial_{\alpha}^{\dot{\alpha}}\bar{\Gamma}_{\beta\dot{\alpha}})\Phi_i\bar{\Phi}^i \\
 (5c) \quad &-igt_1\bar{\theta}^2\mathcal{F}^{\alpha\beta}(\partial_{\alpha\dot{\alpha}}D_{\beta}\bar{D}^{\dot{\alpha}}V)\Phi_i\bar{\Phi}^i \\
 (5d) \quad &t_2\bar{\theta}^2\mathcal{F}^2\bar{\Gamma}^{\alpha\dot{\alpha}}\bar{\Gamma}_{\alpha\dot{\alpha}}\bar{\Phi}_1\bar{\Phi}_2\bar{\Phi}_3 \\
 (5e) \quad &t_3\bar{\theta}^2\mathcal{F}^2\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\Phi_i\bar{\Phi}^i \\
 (5f) \quad &h_{12}\Phi_1\Phi_2\Phi_3 - (h_1 + h_2)\mathcal{F}^{\alpha\beta}\partial_{\alpha}\Phi_1\partial_{\beta}\Phi_2\Phi_3 - \frac{1}{2}h_{12}\mathcal{F}^2\partial^2\Phi_1\partial^2\Phi_2\Phi_3 \\
 (5g) \quad &\bar{h}_{12}\bar{\Phi}_1\bar{\Phi}_2\bar{\Phi}_3 - (\bar{h}_1 + \bar{h}_2)\mathcal{F}^{\alpha\beta}\partial_{\alpha}\bar{\Phi}_1\partial_{\beta}\bar{\Phi}_2\bar{\Phi}_3 \\
 (5h) \quad &\tilde{h}_3\bar{\theta}^2\mathcal{F}^{\alpha\beta}\nabla_{\alpha}\Phi_1\nabla_{\beta}\Phi_2\Phi_3 + \tilde{h}_3\bar{\theta}^2\mathcal{F}^2\nabla^2\Phi_1\nabla^2\Phi_2\Phi_3 \\
 (5i) \quad &h_3\bar{\theta}^2\mathcal{F}^2\nabla^2\Phi_1\nabla^2\Phi_2\Phi_3 \\
 (5l) \quad &h_4^{(=)}\bar{\theta}^2\mathcal{F}^2\nabla^2\Phi_i\Phi_i\bar{\Phi}^i\bar{\Phi}^i; \quad h_4^{(\neq)}\bar{\theta}^2\mathcal{F}^2\nabla^2\Phi_i\Phi_j\bar{\Phi}^i\bar{\Phi}^j \quad i < j \\
 (5m) \quad &h_5\bar{\theta}^2\mathcal{F}^2\Phi_i\bar{\Phi}^i\bar{\Phi}_1\bar{\Phi}_2\bar{\Phi}_3 \quad (\text{A.30})
 \end{aligned}$$

We have not explicitly indicated background or quantum matter fields since it should be clear from the context. For instance, $\Phi_i\bar{\Phi}^i$ stands for $\Phi_i\bar{\Phi}_q^i$ or $(\Phi_i)_q\bar{\Phi}^i$.

We note that all vertices containing quantum gauge fields are at least of order $\mathcal{F}^{\alpha\beta}$. Hence vertices with quantum gauge fields and order \mathcal{F}^2 could be only employed in tadpole

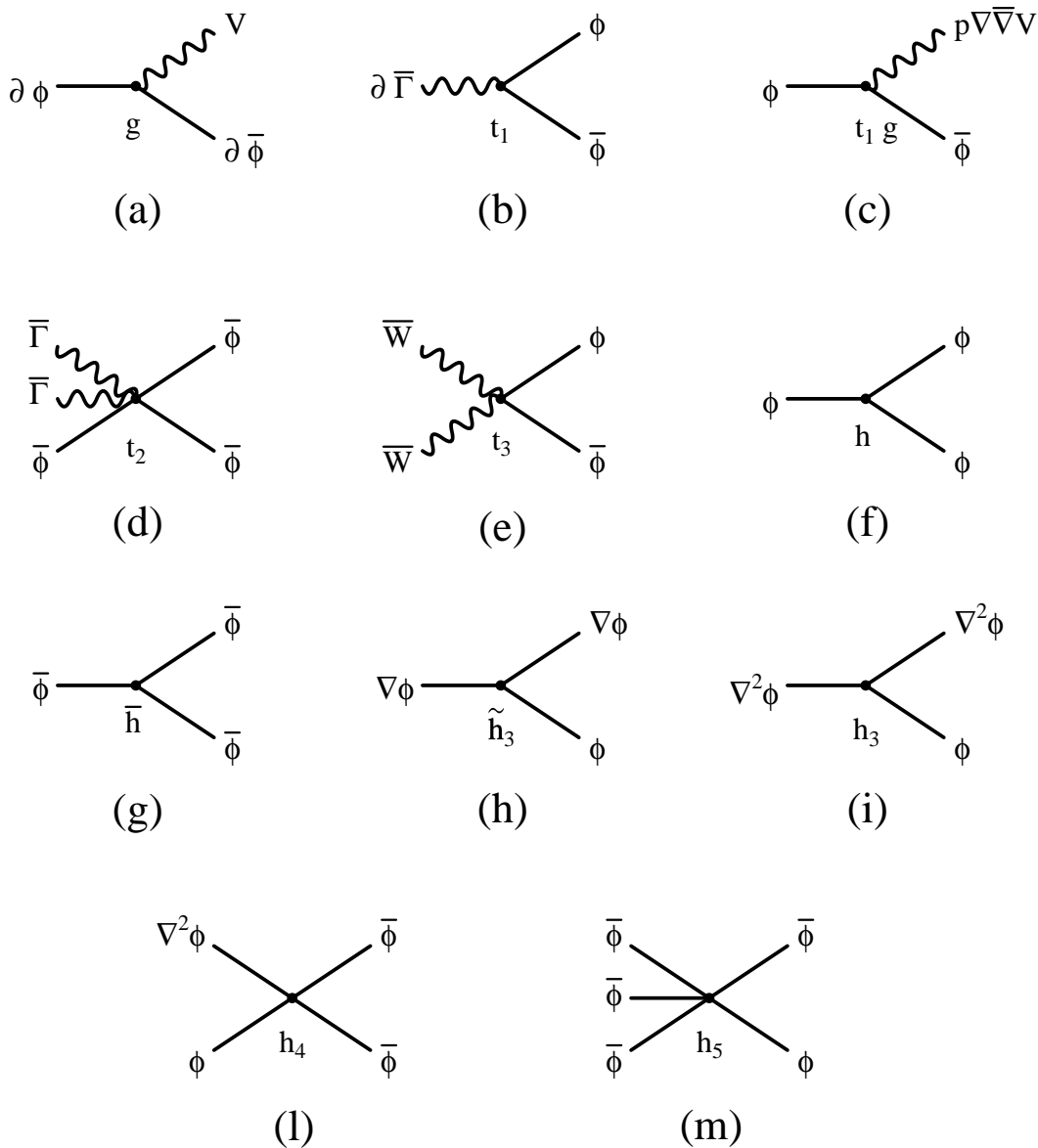


Figure 5. Vertices from the actions (2.12), (2.13).

diagrams which vanish in dimensional regularization. This is the reason why in vertices (5d, 5e) we take gauge fields to be only background.

The expressions for the vertices of the one-flavor case can be obtained from the previous ones by dropping flavor indices and setting

$$\begin{aligned}
 h_1 &= -h_2 = h/2, & \bar{h}_1 &= -\bar{h}_2 = \bar{h}/2 \\
 h_4^{(=)} &= h_4, & h_4^{(\neq)} &= 0
 \end{aligned}
 \tag{A.31}$$

Moreover, we need take into account extra symmetry factors that arise when specifying quantum or background matter. For instance, the term Φ^3 in (5f) would give rise to $3\Phi^2\Phi_q$. The vertex (5h) is absent for trivial symmetry reasons.

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